EXPERIMENT 13

Propagation of Error: An Error Analysis Activity

Objective

More often than not, quantities that are measured are used to determine derived quantities. For instance, mass and volume measurements are used to make density determinations. These measurements have a certain error associated with them. This lab will introduce a method of statistical analysis to propagate error through calculations.

Introduction

Errors of a Calculated Result

If a balance with a ± 0.0001 g error and a pipet with a ± 0.01 mL error are used, how should these errors be combined to determine the error in the density? The use of significant figures is an approximation of error, but for a more exact representation the following methods are used.

Absolute error (Δ) is the approximate error of a single measurement. The absolute error is best estimated as the standard deviation for a measurement. Absent this data an estimation should be made based upon the confidence the experimenter has with their ability to read the measuring device.

Relative error is the ratio of the size of the absolute error to the size of the measurement being made.

$$\text{Relative Error} = \frac{\text{absolute error}}{\text{experimental value}} = \frac{\Delta}{\text{measurement}} \qquad \textbf{(EQ 13.1)}$$

$$\%\text{Relative Error} = \frac{\text{absolute error}}{\text{experimental value}} \times 100\ \% = \frac{\Delta}{\text{measurement}} \times 100\ \% \qquad \textbf{(EQ 13.2)}$$

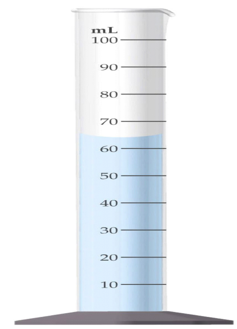
EXAMPLE 13.1

When reading the volume of liquid in the graduated cylinder to the right, you would estimate 64 mL, but it might be 63 or 65 mL. So, the value would be reported as 64 ± 1 mL and the absolute error in the volume measurement, ΔV = 1 mL.

The relative error for the example would be:

$$\text{Relative Error} = \frac{1 \text{ mL}}{64 \text{ mL}} = 0.016$$

or reported as 1.6% relative error.



Using these values we can approximate the error in calculated values. For addition and subtraction the absolute value of the sum or difference can be roughly approximated as the sum of the absolute values.

EXAMPLE 13.2

If the volume of a rock is measured by the displacement of water two volume measurements will be found:

Volume water = 2.5 ± 0.3 mL (range 2.2 - 2.8 mL)

Volume water + rock = 6.8 ± 0.4 mL (range 6.4 - 7.2 mL)

Volume rock = (Volume water + rock) - (Volume water) = 6.8 mL - 2.5 mL = 4.3 mL

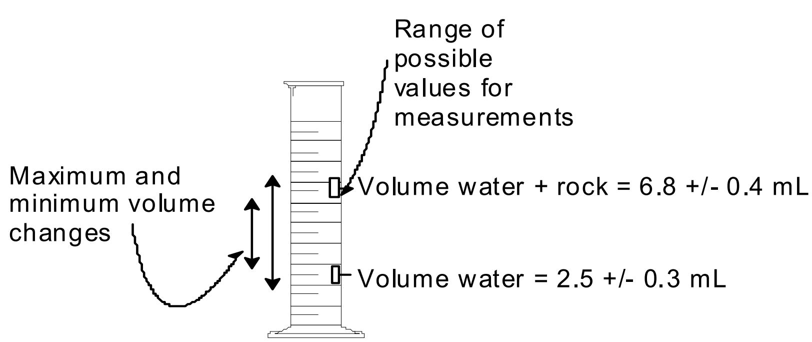
But, the two volume measurements may range over a variety of values. Subtracting the high value for water volume from the low value for water + rock will give a low value for the rock volume:

6.4 mL - 2.8 mL = 3.6 mL

Subtracting the low value for water volume from the high value for the rock volume gives a high volume for the rock volume:

7.2 mL - 2.2 mL = 5.0 mL

The range of values for the rock volume is 3.6 - 5.0 mL or 4.3 ± 0.7 mL. The absolute error in this measurement of 0.7 mL is equal to the sum of the absolute errors for the two original volumes. This idea is illustrated in the picture below.



Volume change = 3.6 to - 5.0mL
(=4.3 +/- 0.7 mL)

This method of error propagation overestimates the combined error because of the possibility that errors can cancel when more than one measurement is made.

In fact, if this process is looked at statistically, a better approximation of error in a sum or a difference is given by the formula:

$$\Delta R = \sqrt{\Delta a^2 + \Delta b^2 + \dots} \qquad \textbf{(EQ 13.3)}$$

When performing multiplication and division, the propagation of error must use relative rather than absolute errors. This is illustrated when density is calculated. The error in mass is in grams, while the error in volume is in mL. Grams and milliliters cannot be added together to calculate the total error; The relative error is unitless. The sum of the relative errors is an approximation of the total relative error (although it is an overestimation as before. Again, if the simple idea of using statistics is applied a better approximation of error results from the following:

$$\text{Relative Error} = \frac{\Delta R}{\mathrm{R}} = \sqrt{\left(\frac{\Delta a^2}{\mathrm{a}}\right) + \left(\frac{\Delta b^2}{\mathrm{b}}\right) + \left(\frac{\Delta c^2}{\mathrm{c}}\right) + \dots} \qquad \textbf{(EQ 13.4)}$$

where R is a calculated value and the final absolute error in the result is given by:

$$\Delta \mathrm{R} = (\text{relative error})(\mathrm{R}) \qquad \textbf{(EQ 13.5)}$$

TABLE 13.3

Operation	**Example**	**Error**
Addition	S = A + B	$\Delta S = \sqrt{\Delta A^2 + \Delta B^2}$
Subtraction	D = A - B	$\Delta D = \sqrt{\Delta A^2 + \Delta B^2}$
Multiplication	$P = A \times B$	$\frac{\Delta P}{P} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$
Division	$Q = \frac{A}{B}$	$\frac{\Delta Q}{Q} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$

EXAMPLE 13.4

A chemist analyzes a compound containing only H, C, and O. What is the uncertainty in the %O? The C-H analysis results are:

%C = 54.80 ± 0.05

%H = 6.92 ± 0.05

Answer:

% O = 100.00% - 54.80% - 6.92% = 38.28%

Applying the rule for subtraction:

$$\Delta O\ \% = \sqrt{(\Delta\%C)^2 + (\Delta\%H)^2} = \sqrt{(0.05) + (0.05)} = 0.07$$

So, %O = 38.28 ± 0.07%

EXAMPLE 13.5

A student finds the density of a liquid by allowing a 10.00 mL volumetric pipet filled with liquid to drain into a previously weighed Erlenmeyer flask. The following data was recorded:

Volume liquid = 10.04 ± 0.01 mL

Mass empty flask = 22.452 ± 0.002 g

Mass flask + liquid = 33.629 ± 0.002 g

Answer:

m = 33.629 - 22.452 g = 11.177 g

V = 10.04 mL

$$d = \frac{m}{V} = \frac{11.177g}{10.04mL} = 1.113\frac{g}{mL}$$

For the error in mass, use the addition rule:

$$\Delta m = \sqrt{(\Delta m_{flask})^2 + (\Delta m_{flask + liquid})^2} = \sqrt{(0.002g)^2 + (0.002g)^2} = 0.003$$

For the error in density, use the rule for division:

$$\frac{\Delta d}{d} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta V}{V}\right)^2} = \sqrt{\left(\frac{0.003g}{11.177g}\right)^2 + \left(\frac{0.01mL}{10.04mL}\right)^2} = 0.035$$

Δd = 0.035 (d)

Δd = (0.035)(1.1113 g/mL) = 0.039 g/mL

However, normally error is only reported to one significant digit because generally speaking, as demonstrated in the above example, error is the least accurate quantity and therefore determines the magnitude of the overall error.

Δd = 0.04 g/mL

Therefore, the student can now report the error in disunity as:

d = 1.12 ± 0.04 g/mL

Note that the final density and the error in the density end at the same decimal place.

See the following web sites for a good discussion of error propagation:

http://www.rit.edu/cos/uphysics/uncertainties/Uncertaintiespart2.html

http://www.itl.nist.gov/div898/handbook/mpc/section5/mcp55.htm

http://teacher.nsrl.rochester.edu/phy_labs/AppendixB/AppendixB.html

Procedure

Density is a physical property of matter. It is defined as the ratio of the mass of an object divided by its volume:

$$d = \frac{m}{V} \qquad \textbf{(EQ 13.6)}$$

For solids and liquids, density is usually expressed in units of g/cm^3. It is evident that the mass and volume of an object must be known to determine its density. A cylindrical shaped metal slug will be used. Its mass will be determined by simple weighing. The volume, however, will be determined by two different methods. An estimate of error will then be made for each method, and then the method or methods with the highest degree of precision will be decided

1. Prepare a data table in your lab book, so that there is space to record the quantities indicated in each part. Be sure to estimate and record the uncertainty of each measurement as you are making the measurements.
2. Obtain the metal cylinder assigned to you. Weigh the cylinder, be sure to record the uncertainty of the measurement.

Vernier Calipers

1. Use Vernier calipers to measure the diameter and length of the metal cylinder to the 0.01 cm. Your instructor will demonstrate how to use these calipers. Make your measurements in centimeters and don't forget to include the uncertainty of your measurements. This will give a volume measurement in cm^3.
2. Using the dimensions of the cylinder determine the volume.

Volume by Water Displacement

1. Partially fill a 50 mL graduated cylinder with water and read the volume. Don't forget to record the uncertainty of the volume measurements. You must initially have enough water in the cylinder to completely cover the metal slug, but be careful that the displaced water does not exceed the 50 mL mark.
2. Tip the graduated cylinder at a sharp angle to the vertical and carefully slide the metal slug into the graduated cylinder. Do not drop the metal slug into the graduated cylinder. Take care to avoid splashing water up the sides of the graduated cylinder, or break the cylinder. Place the graduated cylinder back on your bench and record the new volume. Don't forget to include the uncertainty of the measurement and take the temperature of your water.